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# The Relevance of Multiple Criteria Decision Making to Priorities for Chemical Regulations: An Annotated Bibliography

Tracor Jitco, Inc.
Rockville, MD

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This bibliography contains one hundred and thirteen references and original abstracts on multiple criteria decision making and multi-attribute utility theory. It is a selection of representative articles and reviews rather than an exhaustive review of the literature. This report was prepared in cooperation with the Office of Toxics Integration within the Office of Toxic Substances.

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## ABSTRACT

This bibliography contains one hundred and thirteen references and original abstracts on multiple criteria decision making and multi-attribute utility theory. It is a selection of representative articles and reviews rather than an exhaustive review of the literature. This report was prepared in cooperation with the Office of Toxics Integration within the Office of Toxic Substances.

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# The Relevance of Multiple Criteria Decision Making to Priorities for Chemical Regulations: An Annotated Bibliography

## Introduction

This bibliography contains one hundred and thirteen references and original abstracts on multiple criteria decision making and multi-attribute utility theory. It is a selection of representative articles and reviews rather than an exhaustive review of the literature.

## MAUT

Many articles cited describe multi-attribute utility theory (MAUT), a decision making tool based on describing the utility function of a person or group when several factors (attributes, criteria, variables) are important in determining that function. Selection of advertising campaigns, nuclear power plant sites, and vacation resorts are examples of decisions which have been subjects of MAUT research.

Utility functions are generally discussed in terms of mathematical expressions, although it is not clear that an appropriate and accurate description of a utility function is always possible in mathematical terms.

The utility function is of the form: $U = f(a, b, c, \ldots)$. The number of variables depends on the decision to be made.

Most work in this area addresses discovery of the appropriate variables, and discovery of the form of the utility function, u. Appropriate variables are usually identified through interviews and/or questionnaires involving actual

or surrogate decision-makers. Selecting the form of the function is more difficult. The papers addressing this problem vary from highly theoretical treatments to specific applications. Many demonstrate considerable mathematical complexity.

Most theorists seem to have focused on a simple linear form for u:

$$U = \sum w_i \, s_i$$

In such cases, the $s_i$ are the values (scores) for the i criteria and the $w_i$ are weighting factors which are used to adjust for the varying importance of the different criteria. Appropriate weighting factors are also often discovered by interviews and/or questionnaires.

The primary appeal of the linear form for u is its simplicity. Complex formulations have rarely been shown to predict or describe human behavior any more accurately. The success of the linear formulation seems to derive from the fact that the decisions generally studied are amenable to the necessary assumptions of the linear model. These assumptions, however, limit the applicability of the linear formulations. The most important weakness is that it requires the variables (criteria) to be independent of one another. Unfortunately, many problems do not meet this somewhat artificial requirement. More complex forms of u are tested in several of the papers but the results are often disappointing.

An apparent goal of MAUT and other multiple criteria decision-making research is to reduce the entire decision context to a single expression such as "utility", "go/no-go", or "alternative A is preferable to alternative B". A desire to avoid complexity seems to permeate much of the literature. Perhaps, greater progress could be made by employing partial utility functions and partial results, describing only those interactions of criteria which can be readily discerned. Chipman in his article The Foundations of Utility writes of utility not as a number, but as a vector, the components of which cannot be combined arbitrarily. This may well be a step in the right direction.

## Priority-Setting

Many large organizations such as government agencies need to establish priorities for investment of limited resources. The Environmental Protection Agency - Office of Pesticides and Toxic Substances (OPTS), for example, has oversight responsibility for more than fifty thousand chemicals manufactured, processed, or distributed in the United States. Screening these chemicals and selecting small subsets for closer scrutiny is a difficult task, easily involving enough data to overwhelm human abilities to store, sort, and comprehend. This and similar problems have sometimes been addressed by use of so called "scoring methods or systems". In such a system, a set of relevant criteria is estab- lished, and each member of the set of alternatives (chemicals, in the OPTS example) is scored for each criterion. An overall score for each member is derived from its scores for the various criteria, usually by adding them together. The members can then be ranked according to their overall scores.

Theoretically, scoring systems are very closely related to MAUT and other multiple-criteria decision making models. The main differences arise from practical considerations in priority-setting; the cost of the priority-setting step should not exceed the savings that it will provide. This constraint leads to use of "soft" data and approximate or partial utility functions for deriving overall scores. Scoring systems frequently use a weighted sum of the scores as an overall score, thus making the comparison to classic MAUT all the more striking.

Scoring systems are an obvious approach to problems such as those at OPTS. However, they have often been designed and used without an adequate understanding of their limitations. The fundamental difficulty in applying classical MAUT or scoring methods to priority-setting in chemical regulatory agencies is that independence of the criteria may not be assumed; in fact, synergistic relationships between the criteria is the rule rather than the exception. Therefore, the linear formulations cannot be made to work.

Development of scoring systems is becoming widespread in Federal regulatory agencies. Some large commitments of resources are being made for long-range study of how scoring may be more widely used. An important first step is to study and appreciate what has already been done in the closely related fields covered by these papers. This annotated bibliography was initiated to provide the literature groundwork for what promises to be an exciting effort over the next few years. It was completed by Tracor Jitco Inc., for the U.S. Environmental Protection Agency - Office of Pesticides and Toxic Substances.

Annotated Bibliography

Aschenbrenner KM. 1977. Influence of attribute formulation on the evaluation of apartments by multiattribute utility procedures. In: Jungermann H, de Zeeuw G eds. Decision making and change in human affairs. Dordrecht, Holland: Reidell, p. 81.

The robustness of multiattribute utility (MAU) measures against attribute variations was investigated within the framework of students evaluating off-campus apartment facilities. Two groups of students developed two attribute sets against which descriptions of lodgings were evaluated. The students first intuitively rank-ordered the lodgings according to their impressions of suitability for student housing. They then used two MAU methods for the decomposed evaluations: a riskless simple rating procedure and the risky basic reference lottery ticket. Initial results suggest that the MAU evaluations are not robust against differences in attribute sets; both MAU methods reveal less agreement between groups than the groups' intuitive evaluations. It is noted, however, that elimination of two attributes, monthly and investment costs, yields a much better convergence between the MAU evaluations based on the otherwise different attribute sets.

Aumann RJ. 1962. Utility theory without the completeness axiom. Econometrica 30:445.

A utility theory is developed that parallels the von-Neumann-Morgenstern utility theory, but makes no use of the assumption that preferences are complete, i.e., that any two alternatives are comparable. The von Neumann-Morgenstern utility theory with the completeness axiom is reviewed, and reasons for dropping the completeness axiom are offered. Proofs demonstrating the similarity of the two theories are presented.

Aumann RJ. 1964. Utility theory without the completeness axiom: A correction. Econometrica 32:210.

A correction is presented for two theorems published in another article by this author. The correction is explained for Theorem B and it is noted that a corresponding correction must be made for Theorem C.

Bacharach M. 1975. Group differences in the face of differences of opinion. Manage. Sci. 22:182.

The problems of ranking individual opinions in a group decision situation is addressed. The problem is posed mathematically in an abstract form encompassing a wide range of hypothetical cases, allowing small and big differences over objectives as well as over probability assessments of the facts. General rules are then derived for generating a group ranking for the acts which simultaneously satisfy axioms referring to varying extreme cases. On certain assumptions, the group ranking of acts implies an underlying consensus about the facts. Different axioms of group choice leading to different formulas for consensus are demonstrated. Both positive and negative results are achieved. Under two alternative sets of appealing axioms, the group's implicit consensus is given as a linear opinion pool; that is, the consensual probabilities are weighted averages of the individual ones. On the other hand, under a third set of reasonable axioms, the group ranking cannot be determined by any rules for combining the probabilities separately and combining the utilities. It is concluded that this impossible result depends on wanting the rule to work for cases of extreme differences over ranking of acts.

Bajgier SM et al. 1978. Multiattribute risk/benefit analysis of citizen attitudes towards societal issues involving technology. In: Zionts S, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p. 424.

Factors determining public attitudes and responses to societal issues involving technology are examined. Ten politically sensitive issues, including genetic research, nuclear energy, and the use of saccharin, were presented to 85 college students. Data collected included the perceived economic, environmental, physical, and psychological risks and benefits associated with each of these issues. Multiattribute utility models were developed to predict and represent aggregated perceived risks, benefits, and attitudes regarding these issues. It is felt that results should provide policy makers and researchers with a better understanding of the factors the public considers important and the cognitive processes employed in formulating an overall social attitude or preference regarding these issues.

Bauer V et al. 1977. Applications of multi-attribute utility theory: Comments. In: Jungermann H, de Zeeuw G, eds. Decision making and change in human affairs. Dordrecht, Holland: Reidel, p. 209.

Three papers reviewing the applicability of multiattribute utility theory (MAUT) are discussed. The problem of MAUT model validation is examined, with the conclusion that, except in very simple laboratory experiments, validation of MAUT makes no sense at all. The didactic potential of MAUT is then considered with reference to the problem of multigroup decision making. Next, the absense of real-world applications of MAUT is discussed, and it is concluded that a misconception of MAUT is responsible for its limited use by analysts. It is argued that simple MAUT models are capable of handling complex decision-making problems in the real world.

Bell DE. 1978. Interpolation Independence. In: Zionts S, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p. 1.

The decompositions of multiattribute cardinal utility functions which result from assumptions based on interpolation independence are described. The decompositions discussed involve only single attribute marginal utility functions, together with constants, and are stronger than other decompositions of this type. Complete proofs are not given, but may be obtained from the references.

Blin JM et al. 1978. A multiple criteria decision model for repeated choice situations. In: Zionts S, ed. Multiple criteria decision making. Berlin: Springer-Verlag, p. 8.

Explanations of observed consumer brand switching are reviewed and a multiattribute choice model is proposed which relies on weaker input data than that required by similar models. It is demonstrated that the origin of the brand-switching phenomenon can be explained by the interaction of two factors: (1) multiplicity of evaluative dimensions for the brands; and (2) consumer uncertainty over salient attributes and product performance on these attributes. The analysis also integrates two apparently divergent views of consumer choice: the deterministic view which has evolved around multiple attribute choice models and the stochastic view which has been proposed to

explain brand switching. It is concluded that, far from being a random element superimposed over an intrinsically multiattributed evaluation process, brand switching appears in the model as a perfectly rational strategy for a consumer faced with imperfect knowledge about multiattributed brands. The applicability of the methodology to other existing multiattribute choice models is briefly discussed.

Bolinger JJ et al. 1978. Decision analysis utilizing multi-attribute utility theory in engineering evaluation.
IEEE Trans. Power Appar. Sys. PAS-97:1245.

The application of decision analysis using multiattribute utility theory to a power system engineering evaluation is demonstrated through an example problem involving the evaluation of streetlight expansion options in a regulated utility. The streetlight example problem is defined and then presented in terms of a decision tree. The methodologies of establishing the main and subobjectives, determining the single attribute utility functions, quantifying the preference trade-offs, and calculating the scaling constants are illustrated. A detailed problem analysis is also performed, including expected value and sensitivity analyses. The multiattribute decision analysis is compared with evaluations that consider only a single decision criterion, and the advantages of the multiattribute analysis are documented. It is concluded that this theory allows a decision maker to account for impacts on all relevant areas when making a decision, and results in a completely documented, sound decision. Examples where multiattribute utility theory can be used are cited.

Bowman EH. 1963. Consistency and optimality in managerial decision making.
Manage. Sci. 9:310.

Some research, ideas, and theory about managerial decision making are reported. Aggregate production and employment scheduling are discussed with specific reference to the Holt, Modigliani, Muth, and Simon (HMMS) decision rules. The idea that management's own past decision can be incorporated into a system of improving present decisions is proposed. Decision rules are developed, with the coefficients in the rules derived from management's past

decision rather than from a cost or value model. Half a dozen test cases illustrate and test these ideas. Some rationale about decision making in organizations and criteria surfaces is supplied to help interpret the major ideas presented.

Charnetski JR et al. 1978. Multiple-attribute decision making with partial information: The comparative hypervolume criterion.
Naval Res. Log. Quart. 25:279.

An approach is presented for analyzing multiple-attribute decision problems in which the set of actions is finite, the utility function is additive, and the decision maker is operating with only partial information about the situation. The construction of a set of feasible weight vectors for the various criteria from an ordinal rating by the decision maker is discussed. The Comparative Hypervolume Criterion (CHC) is introduced for choosing an action when the set of feasible weight vectors is not a unique point, and the Monte Carlo method developed for accurately approximating the numerical quantities used by the CHC is described. The numerical method is summarized and a simple example is provided to evaluate the efficiency of a computer code developed to implement the method.

Charnetski JR et al. 1979. Multiple-attribute decision making with partial information: The expected-value criterion.
Naval Res. Log. Quart. 26:249.

The multiple-attribute decision problem with finite action set and additive utility function is considered. It is assumed that the decision maker cannot specify nonnegative weights for the various attributes which would resolve the problem, but that he supplies ordinal information about these weights which can be translated into a set of linear constraints restricting their values. A bounded polytope of feasible weight vectors is then determined. It is further assumed that each element of the bounded polytope has the same chance of being the "appropriate one." The expected utility value of each action is computed on that supposition. The computation method used a combination of numerical integration and Monte Carlo simulation and is deemed equivalent to finding the center of mass of the bounded polytope. Comparisions are made with the comparative hypervolume criterion and two numerical examples are presented.

Chipman JS. 1960. The foundations of utility.
Econometrica 28:193.

The mathematical foundations of rational behavior, in the sense of a transitive ordering of alternatives, are developed without making any assumptions about the special character of the set of alternatives from which choices are made. It is shown that the ordering of alternatives may be characterized by a utility function, where utility is represented by a vector with real-valued components, such vectors being ordered lexicographically. It is stated that if an axiom permitting comparison of intensities is permitted, such a utility index must be unique up to transformation (such as proportionality transformations) preserving group operations. A purely topological axiom, called the axiom of substitution, is shown to imply that utility is real-valued, i.e., that the above vector has only one component.

Churchman CW et al. 1954. An approximate measure of value.
J. Ops. Res. Soc. Am. 2:172.

A method for estimating importance on a scale of value is presented, based on actual or verbal choices of decision makers. It is felt that the method can be applied to any number of objectives or outcomes, and to any number of decision makers. It is observed that measures of the reliability of the estimates can be obtained, but that the method does not provide any estimate of the accuracy or bias of the judgments.

Dalkey NC. 1976. Group decision analysis. In: Zeleny M, ed. Multiple criteria decision making. Kyoto 1975. New York: Springer-Verlag, p. 45.

Group decision making is investigated. It is shown that consistent group preference functions can be formulated by the use of anchored scales, i.e., individual preference scales with fixed reference objects. It is observed that no general resolution of the aggregation problem for probabilities appears feasible, but that a justification for the use of group probability judgments can be made based on a family of theorems to the effect that the accuracy of a group judgment is always greater than (or, at worst, equal to) the average accuracy of the individual judgments. Some empirical data and analytical results are felt to indicate that these aggregation rules are more generally applicable and more powerful than has been assumed in the past.

Dawes RM. 1971. A case study of graduate admissions: Application of three principles of human decision making.
Am. Psychol. 26:180.

Three major principles are discussed that have been determined to be applied to the decision making problems of evaluating graduate school applicants. As a case study of the admission decision of one university's psychology department, the work is concerned with the decision making process rather than with the validity of the criteria on which decisions are based. The three principles examined are: (1) that a simple linear combination of the criteria considered by the admissions committee will predict performance in graduate school better than the committee; (2) that the behavior of the committee can be simulated by a linear combination of the criteria it considers; and (3) that the paramorphic representation of the committee's judgements should be more valid than the judgments themselves. Multiple regression analysis was performed using four variables commonly used for evaluating applicants. It was found that a paramorphic representation of the admissions committee's judgments predicts the first-year evaluation better than does the judgment itself. A cost-benefit analysis of the method is presented, and the ethics of using a regression analysis in this situation are considered.

Diamond PA. 1965. The evaluation of infinite utility streams.
Econometrica 33:170.

Impatience implications of several axiom sets assumed for preferences over an infinite future are explored. Sufficient conditions for the existence of a continuous utility function are presented. This analysis is done both for the product topology and the metric function equating the distance between two infinite streams to the maximal one-period difference between them.

Dyer JS. 1973. A time-sharing computer program for the solution of the multiple criteria problem.
Manage. Sci. 19:1379.

A description is presented of a time-sharing computer program written to implement a man-machine interactive algorithm for the solution of the multiple criteria problem. It is stated that a unique feature of this program is the

man-machine dialog which obtains information from the decision maker through a series of simple, ordinal comparisons. This procedure requires the decision maker to specify his local marginal rates of substitution among criteria and to solve a step-size problem at each of several interations. The computer program was written in APL and implemented on an IBM 360-91 time-sharing system. It may be used successfully by untrained individuals to solve problems involving multiple criteria.

Edwards W. 1977. How to use multiattribute utility measurement for social decision making.
IEEE Trans. Syst. Man Cybern. SMC-7:326.

Multiattribute utility measurement is proposed for determining individual or group values, showing how and how much they differ and how the extent of such differences can be reduced. Ten steps concerning the use of multiattribute utility measurements are explained. Data are then presented illustrating the application of this technique to three situations: land use regulation by the California Coastal Commission, selection of government research programs, and development of a water quality index. A different approach was used in each case. It is felt that face-to-face discussion was most successful in producing agreement, while procedures depending on written or verbal feedback of other expert's judgments were less so. The importance of simplicity in any elicitation procedure is discussed. It is concluded that exploitation of multiattribute utility measurement permits a decision making agency to shift attention from the specific actions being regulated to the values these actions serve and the decision making mechanisms that implement these values.

Einhorn HJ. 1971. Use of nonlinear, noncompensatory models as a function of task and amount of information.
Org. Beh. Human Perform. 6:1.

The effect of the two variables of type of decision task and amount of information was assessed as the two variables affect the use of nonlinear, noncompensatory models in decision making. The former variable was found to have a marked effect on the kind of combination model used by the subjects,

while the latter variable had a significant effect on the accuracy with which the various models could adequately represent the decision process. The major findings are deemed to show considerable use of the nonlinear, noncompensatory models with differential use in the two decision tasks used. It is felt that results for amount of information suggest that subjects use complex combinations of models (or compound models) to simplify the situation cognitively. In addition, it is felt that the claim that decision makers do not use additive models in their strategies is given support by these data.

Einhorn HJ. 1972. Expert measurement and mechanical combination. Org. Beh. Human Perform 7:86.

The superiority of mechanically combining components as opposed to using the global judgment for predicting some external criterion is discussed. The use of components is extended to deal with multiple judges since specific judges may be differentially valid with respect to subsets of components for predicting the criterion. These ideas are illustrated using results of a study dealing with the prediction of survival on the basis of information contained in biopsies taken from patients having a certain type of cancer. Judgments were made by three highly trained pathologists. Implications and extensions for using expert measurement and mechanical combination are discussed.

Einhorn HJ et al. 1978. A simple multiattribute utility proceedure for evaluation. In: Zionts S, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p. 87.

A simple multiattribute utility procedure (SMAUP) is developed within the context of the evaluation of overall performance of players in the National Basketball Association (NBA). The specific simplifications assume that: (1) utility functions for the attributes are linear; (2) the total utility of an object is an additive function of the utility of the attributes making up that object; (3) the variables are scaled so that they are monotonically increasing in utility; (4) the relative importance of the attributes is correctly identified and no important attributes have been omitted; and (5) the attributes are essentially uncorrelated with each other. Results are compared

with an independent measure of overall performance (viz., the players picked for the NBA all-star team), and the advantages of SMAUP versus clinical judgement are discussed. Generally, it is felt that SMAUP is a useful procedure in any situation where the above assumptions are considered reasonable.

Farquhar PH. 1974. Fractional hypercube decompositions of multiattribute utility functions. Tech. Rep. 222, Dept. of Oper. Res., Cornell Univ., Ithaca, N.Y.

Some new results in multiattribute utility theory are presented, showing how fractional hypercubes induce the multiple element conditional preference orders used to specify attribute independence conditions and how they also provide a system of equations used to produce the corresponding multiattribute utility decomposition. Fractional hypercube decompositions incorporating the additive, quasi-additive, and diagonal forms are reviewed; then, decompositions with nonseparable attribute interactions are discussed. Several special cases examined in detail give decompositions with nonseparable lower order interactions and separable higher order interactions. Other examples illustrate hybrid decompositions which combine different independence conditions in one form. A thorough discussion is given of the various types of nonseparable and separable interactions appearing in fractional hypercube decompositons. Several examples and potential applications to real decision problems are provided.

Farquhar PH. 1975. A fractional hypercube decomposition theorem for multiattribute utility functions.
Oper. Res. 23:941.

A general method is presented for producing independence conditions for nonadditive utility decompositions based on geometric structures called fractional hypercubes. A fractional hypercube is defined and the corresponding multiple element conditional preference order is introduced. The main theorem is produced from the solution of equations that are derived from transformation of linear functions that preserve these conditional preference orders. The computations and scaling required in implementing the

main result are demonstrated by obtaining four utility decompositions on three attributes: apex, diagonal, quasi-pyramid, and semicube. The methodology is illustrated with geometric structures that correspond to the fractional hypercubes. Because this method is so general and since fractional hypercubes illustrate the interactions they ultimately produce in a decomposition, it is argued that an analyst will be able to tailor his independence conditions and utility decomposition to a particular decision problem rather than be restricted to a few nonadditive decompositions.

Farquhar PH. 1976. Pyramid and semicube decomposition of multiattribute utility functions.
Oper. Res. 24:256.

The fractional hypercube decomposition theorem for multiattribute utility functions is applied to the quasi-pyramid, the pyramid, and the semicube fractions. The resulting nonadditive utility models illustrate the broad range of decompositions that can be obtained with this fractional hypercube methodology. The pyramid decompositions have nonseparable interaction terms for each pair of attributes, whereas the semicube decomposition has nonseparable interaction terms for each proper subset of attributes. Although the semicube decomposition is much more general than the pyramid decomposition, it is suggested that the latter is simpler to implement in practical problems. It is noted that the illustrated computations and scaling can be performed with other fractional hypercubes to produce various results between these two extremes. A condensed version of the fractional hypercube methodology is presented.

Farquhar PH. 1977. A survey of multiattribute utility theory and applications.
TIMS Studies in the Management Sciences 6:59.

An extensive survey is presented of the state-of-the-art in multiattribute theory and applications. Particular attention is given to recently developed utility models for evaluating risky decision alternatives when the attributes describing the outcomes of these possible decisions may be interdependent. Nonrisky and risky additive utility models are discussed, as well as

multiplicative and quasi-additive utility models and extensions, bilateral and multilateral utility models, fractional hypercubes, interdependent additivity, parametric utility models, approximation methods, and trade-off analysis. Some practical applications of multiattribute utility models are reviewed and directions for future research are suggested. A brief overview of the terminology and results of expected utility theory also is offered.

Farquhar PH. 1978. Interdependent criteria in utility analysis. In: Zionts s, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p. 131.

Decision problems are examined where the evaluative criteria are interdependent. After a brief study of interaction effects in the statistical analysis of multifactor experiments, a classification of interactions is developed for multiple criteria decision problems. The classification includes artificial, ordinal, configural, and holistic interactions. It is shown that some interactions are removable by transforming the response scale or restructuring the factors, while other interactions are not removable and must be considered explicitly. The interaction effects in various nonadditive utility models are discussed, and the method of fractional hypercubes is used to interpret the structrual forms of these models.

Fisher GW. 1972. Four methods for assessing multi-attribute utilities: An experimental validation. Tech. Rep. 037230-6-T. Ann Arbor, Mich: Engineering Psych. Lab., Univ. Mich.

Decomposition evaluation procedures are investigated to determine their superiority over holistic intuitive judgements. The theoretical basis for multiattribute value assessment is first described and it is concluded that while additive evaluation models are appropriate for most riskless decisions, nonadditive models are frequently required for decision making under uncertainty. The sensitivity of evaluation models to assessment errors is then discussed. Next, four procedures are described for constructing a decomposed evaluation model and the general problem of validating evaluation procedures is considered. Finally, two experiments are presented to demonstrate that the four decomposition procedures described can provide an appropriate measure of subjective value.

Fischer GW. 1975. Experimental applications of multi-attribute utility models utility probability and human decision making. In: Wendt D, Vlek CAJ, eds. Dordrect, Holland: Reidel, p. 7.

Experimental studies attempting to model the subjective trade-off process involved in decision making are reviewed. Literature devoted to the goal of developing procedures which can be used to improve the subjective evaluation process also is reviewed. Formal models for both riskless and risky choice are discussed. Some practical scaling procedures are briefly outlined, and results from experimental attempts to validate these scaling techniques are presented. The superiority of non-additive evaluation procedures to additive evaluation methods is also considered.

Fischer GW. 1977. Convergent validation of decomposed multi-attribute utility assessment procedures for risky and riskless decisions. Org. Beh. Human Peform. 18:295.

The relative merits of holistic and decomposed assessment procedures are considered, and decomposition procedures for both risky and riskless decision making are described. Next, the validity of evaluation functions for both procedures is assessed. Conjoint and function measurement are considered for the holistic evaluation method, and convergent or conjoint validity is examined for the decomposition approach. Finally, an experiment comparing holistic and decomposed assessment procedures is discussed in which the two approaches to scaling were found to assign very similar utilities to three-dimensional job descriptions for both risky and riskless decisions.

Fishburn PC. 1965. Independence in utility theory with whole product sets. Oper. Res. 13:28.

Because it is felt that a satisfactory definition of independence based on multidimensional consequences and hypothetical gambles composed of such consequences is lacking, a definition of independence is presented for cases where the set of consequences X is a product set $(X_1)\ (X_2)\ldots(X_n)$, each element in X being an ordered n-tuple $(x_1, x_2,\ldots,x_n)$. The definition is stated in terms of indifference between special forms of gambles formed from the set of consequences. It is then shown that if the condition of the

definition holds, the utility of each $(x_1, x_2, \ldots, x_n)$ in X can be written in the additive form $u(x_1, x_2, \ldots, x_n) = u(x_1) + u(x_2) + \ldots + u(x_n)$, where u is a real-valued function defined on the $X_i$, i = 1, 2, ..., n. Uniqueness properties for the individual utility functions are also offered. The development is carried out for two factors and then for n-factors. It is noted that the development is free from any specific assumptions about the utility function (e.g., continuity, differentiability), except that it be a von Neumann-Morgenstern utility function, and that it places no restrictions on the natures of the individual factors. An example is offered to illustrate one type of situation with which the theory is concerned, and some comments are given about experimentation and extension to the case where the set of consequences X is a proper subset of $(X_1)(X_2)\ldots(X_n)$.

Fishburn PC. 1965. Independence, trade-offs, and transformation bivariate utility functions.
Manage. Sci. 11:792.

Interrelationships between independence in the utility sense, trade-off or indifference curves, and transformation curves are examined with respect to bivariate utility functions, i.e., those defined on rectangular regions in the xy-plane and on 50-50 gambles formed from points in such regions. Following the form in which a set of basis axioms of utility are stated, independence is defined in terms of indifference between 50-50 gambles. It is shown that, if the condition of the definition holds, the bivariate utility function of (x, y) can be written in the additive form. The concepts of trade-off curve and transformation curve are also defined on the basis of the indifference relation but are seen as not concerned with gambles. After exploring relationships among the three notions, it is shown how the utility curves for the two variables under independence can be constructed on the basis of two trade-off curves.

Fishburn PC. 1965. Markovian dependence in utility theory with whole product sets.

Oper. Res. 13:238.

The notion of Markovian dependence in utility theory is developed, considering an entire product set as the basic set of consequences. Markovian dependence in the utility sense is defined on the basis of indifference between special types of gambles for several sequential processes. Theorems stating how the basic utility function may be broken up into additive forms under Markovian dependence are presented. In addition, the notion of stationary transition value mechanisms is explored in the context of completely homogeneous sequential processes.

Fishburn PC. 1966. Additivity in utility theory with denumerable product sets.

Econometrica 34:500.

The expression of utility in the additive form is extended to the denumerable-set context. The extension is accomplished by a gross axiom containing three basic parts: monotonicity, additivity, and convergence. The axiom is then interpreted in three settings for utility: gambles or lotteries, utility differences, and binary-choice probabilities.

Fishburn PC. 1966. A note on recent developments in additive utility theories for multiple-factor situations.

Oper. Res. 14:1143.

Developments in additive utility theories are summarized for research workers interested in the subjective evaluation of multiple-factor alternatives. In particular, assumptions (axioms) that permit the use of an additive-value formula are presented. It is felt that these may enable the user to test the warrantability of an additive procedure in specific situations where the theory might be applied.

Fishburn PC. 1967. Interdependence and additivity in multivariate unidimensional expect utility theory.
Int. Econ. Rev. 8:335.

Additivity in expected utility theory is investigated in the presence of interdependencies among different coordinates when the set of pure prospects is in the Cartesian product of n-other sets. A special case of the theory is mentioned which deals with additivity in the expected utility context when there are no interdependencies among the coordinates. The additivity theorem is developed and its proofs are given. Admissible transformations also are specified for the expected utility function.

Fishburn PC. 1967. Methods of estimating additive utilites.
Mange. Sci. 13:435.

Additive utility formulations for risky and nonrisky multiple-factor decision situations are reviewed. Twenty-four methods of estimating additive utilities are listed and classified. References to the theory and technique of each method are given along with a short discussion of each. A number of examples are used to illustrate the methods.

Fishburn PC. 1968. Utility theory.
Manage. Sci. 14:335.

The concept of utility is discussed, explaining why people are interested in it and how it is interpreted and used in the management and behavioral sciences. Several utility theories are summarized and a semi-technical survey of particular theories is presented for readers interested in greater depth. A prescriptive interpretation of utility theory is emphasized.

Fishburn PC. 1970. Utility theory with inexact preferences and degrees of preference.
Synthese 21:204.

Various properties of the strength-of-preference comparison relation are examined along with properties of simple preferences defined from the relation. The investigation recognizes an individual's limited ability to make precise judgments. The relationship between the concept of comparable preference differences and ordinal, cardinal, and measurable utility also is considered.

Fishburn PC. 1973. A mixture-set axiomatization of conditional subjective expected utility.
Econometrica 41:1.

An axiomatization is presented for a states-of-the-world-type conditional subjective expected utility model. The axioms consist of extensions of previously derived axioms for measurable utility, a generalization of an averaging condition, and several structural conditions. The structural conditions are examined in some detail, and examples are given showing what happens to the numerical model when the conditions do not hold. The numerical model is found to express the utility of an act, given an event, as a weighted sum of the utilities of the act given events that partition the initial event, the weights being personal probabilities for the partition events conditioned on the initial event. The theory is compared to other theories for conditional expected utility.

Fishburn PC. 1973. Bernoullian utilities for multiple-factor-situations. In: Cochrane JL, Zeleney M, eds. Multiple criteria decision making. Columbia, S.C. Univ. of S.C. Press, p. 47.

Under the assumption that an individual has a von Neumann-Morgenstern or Bernoullian utility function u on a product set $X_1 \times X_2 \times \ldots \times X_n$ (with gambles defined on this set on n-tuples), preference conditions are discussed which are necessary and sufficient for a special form for u which involves sums and products of functions on the several $X_i$ factors. It is noted that the preference conditions for this form specify a certain type of consistency among the preference orders over gambles defined on one factor for various pairs of fixed n - 1 tuples of values from other n - 1 factors. The special form is presented in the three-factor case. A two-period income-stream example then illustrates the potential applicability of the two-factor case when simpler forms cannot be used for u. The general theory for the n-factor case and a sufficiency proof are given and aspects of scaling procedures for the functions and constants of the theorem are discussed.

Fishburn PC. 1976. Conjoint measurement in utility theory with incomplete product sets.
J. Math. Psychol. 4:104.

A set of axioms is introduced for additive conjoint measurement in utility theory when the set of consequences is a subset of the Cartesian product of two sets. Axioms for preference, patterned after the von Neumann-Morgenstern axioms, are presented first. These axioms imply the existence of a utility function on the set of consequences and on gambles formed from the elements in that set that is unique up to a positive linear transformation, but they do not imply the additive form of conjoint measurement. It is then shown that the addivite form results when one of the basic axioms is strengthened, so that the utility of each consequence equals the sum of the utilities of its two factors. Uniqueness properties of the factor utilities are also discussed.

Fishburn PC. 1977. Approximations of two-attribute utility functions.
Math Oper. Res. 2:30.

Von Neumann-Morgenstern utility functions are assessed from the viewpoint of mathematical approximation theory since it is felt that the independence conditions that characterize two-attribute and other special forms of utility functions may be inapplicable because of evaluative interdependencies among the attributes. The assessment focuses on approximations v of the utility function u that can be written as $v(x,y) = f_1(x)g_1(y) + \ldots + f_m(x)g_m(y)$ where $f_1$ and/or $g_1$ are based on single-attribute conditional utility functions. It is observed that the paper does not rely on independence axioms that enable u(x,y) to be expressed in simplified forms when they are valid.

Fishburn PC. 1977. Multiattribute utilities in expected utility theory. In: Keeny RL, Raiffa H, eds. Conflicting objectives. New York: Wiley, p. 172.

The problem of assessing multiattribute utilities within the context of a von Neumann-Morgenstern utility function on a multiattribute utility space is examined. Possibilities for simplifying multiattribute utility assessment are investigated, with emphasis on the idea of expressing the n-attribute utility function as some combination of conditional single-attribute utility

functions. To this end a sequence of increasingly sophisticated independence conditions is considered, beginning with the concept of value independence among attributes. Also examined are aspects of the concepts of utility independence and generalized utility independence, which, in appropriate conditions, are observed to lead to additive, multiplicative, or quasi-additive forms for the utility function. This is followed by a discussion of the concept of bilateral independence, which can accommodate more interdependencies among attributes but still allows the utility function to be written in terms of conditional single-attribute utility functions. The discussion of independence concludes with a general view of fractional independence that involves numerous special cases, including utility independence and bilateral independence. The final section, which is viewed as not predicated on any independence conditions, considers the use of conditional single-attribute utility functions in constructing approximations of arbitary n-attribute utility functions.

Fishburn PC. 1978. A survey of multiattribute/multicriterion evaluation theories. In: Zionts S, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p. 181.

Multiattribute/multicriterion evaluation theories are reviewed. The general formulation for the survey is presented first, followed by a discussion of classificatory attributes of the theories. Later sections are organized around a three-valued attribute whose values correspond to the familiar categories of decision under certainty, decision under risk, and decision under uncertainty. Reference is occasionally made to various choice models, but the emphasis is on evaluation theories rather than assessment methodologies in the main body of the paper. The final section presents a brief review of assessment literature.

Fishburn PC et al. 1967. Additive utilities with incomplete product sets: Application to priorities and assignments.
Oper. Res. 15:537.

Additive utilities are investigated for cases where the set of consequences is a subset of the product of n-other sets, i.e., an incomplete product set. It was found that, under a finiteness restriction, an additivity

axiom leads to an additive utility representation for the elements in the set of consequences. The potential usefulness of the theory in connection with incomplete product sets is demonstrated with priority orderings and assignments.

Fishburn PC. 1974. Lexicographic orders, utilities and decision rules: A survey.
Manage. Sci. 20:1442.

Recent developments in lexicographic orders, preferences, utilities, probabilities, and decision rules are reviewed and some new results are presented. Two trends in the literature on lexicographic orders are identified: the pragmatic and the mathematical. For the former, two axiomatizations for lexicographic preferences that avoid direct reference to the importance ordering of criteria are offered, along with comments on additive utilities, dictators in social choice theory, and an example which illustrates the failure of the axiomatizations when the number of criteria is infinite. Modifications on the basic lexicographic idea are discussed that are designed to make the lexicographic approach more palatable in studies of normative decision making and choice behavior. Focus is then shifted to the mathematical question of whether a preference order can be represented by a real-valued function and, if not, what sort of lexicographic representation might apply. Necessary and sufficient conditions for unidimensional utility are considered. Examples, including several in consumer choice theory, are presented, as well as some related material for mapping into real intervals. The idea of lexicographic representations in structures that possess some type of additive operation is examined. Examples of lexicographic probability representations which cannot be characterized by a single probability measure are presented. Finally, lexicographic expected utility is discussed.

Fishburn PC. 1974. von Neumann-Morgenstern utility functions on two attributes.
Oper. Res. 22:35.

Five special forms for a von Neumann-Morgenstern utility function on a product set (A times X) are examined. Each form expresses the utility function as a combination of functions defined on the separate factors of the

product set. The five forms include the additive and the multiplicative forms and the additive form with independent product. Necessary and sufficient preference conditions are indentified for each form, along with admissible transformations on the single-argument functions.

Fishburn PC. 1975. Unbounded expected utility.
Annals. Stat. 3:884.

Axioms are presented for a preference relation on P, a convex set of finitely additive probability measures defined on a Boolean algebra of subsets of a set X of consequences. The axioms are necessary and sufficient for the existence of a real-valued utility function u on X for which p is preferable to q if and only if E(u,p) is greater than E(u,q), for all p and q in P. A slightly simpler set of axioms is found to yield the same results when the algebra is a Borel algebra and every measure in P is countably additive. The axioms allow P to contain nonsimple probability measures without necessarily implying that the utility function is bounded.

Fishburn PC et al. 1974. Seven independence concepts and continuous multiattribute utility functions.
J. Math. Psychol. 11:294.

Seven independence concepts are examined based on a preference relation on the set of simple probability measures defined on a set of multiattribute consequences. Three of the independence relations involve gambles and the other four are based on riskless preferences over the n-tuples in the consequence set. The main theorems state conditions under which one or more of the risky independence relations can be derived from a riskless independence relation in conjunction with other conditions. The other conditions include a risky independence condition which differs from the ones derived, the assumption that the consequence set is a convex subset of a finite-dimensional Euclidean space, and the assumption that the individual's von Neumann-Morgenstern utility function on the consequences set is continuous.

Fishburn PC et al. 1975. Generalized utility independence and some implications.
Oper. Res. 23:928.

The concept of generalized utility independence is introduced and three functional forms for a multiattribute von Neumann-Morgenstern utility function are derived, subject to various generalized utility independence assumptions. The three forms are the additive, the multiplicative, and the quasi-additive, each of which expresses the multiattribute von Neumann-Morgenstern utility function as a combination of utility functions defined on the separate attributes. It is demonstrated that if the utility function is unbounded from above and below, then given the three forms, either reversal of preference over some attributes occurs or else the additive form must hold.

Giauque WC et al. 1976. Application of multidimensional utility theory in determining optimal test treatment strategies for streptococcal sore throat and rheumatic fever.
Oper. Res. 24:933.

A model is presented for determining optimal test and treatment strategies for streptococcal sore throat and rheumatic fever, in which the formulation and assessment of a multidimensional utility function are a central part. Unanimity of opinion regarding the proper course of medical care implied by the utility theory is explained by the overriding importance of the catastrophic possibilities of either severe antibiotic reaction or the long-term effects of rheumatic fever. As the probability of experiencing a catastrophic antibiotic reaction was the lesser of the two, the option of using penicillin fairly liberally was dominant. Model results also confirmed the propriety of continued prophylactic penicillin treatment and of routine community-wide streptococcal screenings. Finally, the relative effectiveness of a physician was compared to that of a nurse-practitioner in the diagnosis and care of streptococcal infections, with the result that the nurse was shown to be at least as effective as the physician. It is concluded that the complexity and time demands of the analysis probably preclude patient-by-patient analysis, but the methods could assist in determining practices for dealing with entire groups of patients with specific groups of symptoms.

Goldberg LR. 1968. Simple models or simple processes? Some research on clinical judgements.

Am. Psychol. 23:483.

Because the validity of clinical judgments has been questioned, results are presented of investigations which focused on the process of clinical inference, the aim of the investigations being to simulate the hidden cognitive processes of the clinician as he makes judgmental decisions. Multiple-regression analysis of simple linear models and fixed-model analysis of variance are examined and it is concluded that, although judges can process information in a configural way, the general linear model is powerful enough to reproduce most of these judgments with very small error. Results are then discussed of attempts to improve the accuracy of judgments of practicing clinicians. The hypothesis that receiving immediate feedback would increase a clinician's judgmental accuracy was not verified. It is felt that a good deal more than outcome feedback is necessary to accomplish this goal.

Goldberg LR. 1971. Five models of clinical judgement: An empricial comparison between linear and nonlinear representations of the human inference process.

Org. Beh. Human Perform. 6:458.

Two nonlinear models which had been proposed in previous research as approximations of the conjunctive and disjunctive strategies were compared with the linear, logarithmic and exponential models as potential representations of the judgments made by 29 clinical psychologists, each of whom attempted to differentiate neurotic from psychotic patients on the basis of their MMPI profiles. For this task the linear model was found to provide a better representation of the judgments made by all clinicians than did either of the nonlinear models, and only the logarithmic model provided the linear model with any real competition. In addition, it is felt that the serious problems endemic to all of the nonlinear models call into question their utility as potential judgmental representations.

Gorman WM. 1968. Symposium on aggregation: The structure of utility functions.
Rev. Econ. Stud. 35:367.

A continuous utility function is defined and a theorem stating the separability and essentiality of two sets is proved. A decision tree is developed and results are applied to the theory of decision over time and to the aggregation of production relations. An alternative approach is briefly discussed that is felt to be easier to apply in the case of complete additive separability. The results of relaxing the condition of strict essentiality are sketched.

Green PE et al. 1972. Subjective evaluation models and conjoint measurement.
Behav. Sci. 17:228.

The predicted effectiveness of self-explicated models and empirically derived evaluation functions is compared. Several types of measurement and scaling models are applied to empirical data involving a sample of housewives' preferences for profile descriptions on 27 hypothetical retail discount cards. Additive conjoint measurement is used to obtain part-worth functions for the various component descriptors. In addition, subjects' self-explicated models are compared with actual ranking of the discount cards. Results indicate that additive conjoint models provide good descriptions of the data and that self-explicated models also show reasonable accuracy in predicting actual ranking. No significant association was found between differences in evaluative behavior and a battery of attitudinal and socioeconomic measurements obtained from the same subjects. The compensatory type of subjective evaluation model used in this study is described, and conjoint measurement is briefly explained.

Green PE et al. 1973. Consumer evaluation of discount cards: A conceptual model and experimental test.
J. Retailing 49:10.

The application of conjoint measurement to a study of consumer evaluation of discount card characteristics is illustrated. The principles of conjoint measurement and its uses in consumer attitude research are briefly described; then, a particular case of conjoint measurement, namely the additive model, is

applied to measure consumer evaluations of the benefit-cost profiles of retail discount cards. Findings indicate that simple main-effects conjoint model can provide a good account of the data and that a more complex interactive-type model is not required in this type of experiment. Subjects' tendencies to exhibit more uniform self-explicated weights than the part-worth weights developed from overall preference responses are also analyzed.

Hauser JR et al. 1979. Assessment of attribute importances and consumer utility functions: von Neumann-Morgenstern theory appled to consumer behavior. J. Consumer Res. 5:251.

The applicability of von Neumann-Morgenstern utility theory to modeling consumer choice behavior is assessed. Key concepts of utility theory that are important to consumer behavior are reviewed and the differences between von Neumann-Morgenstern utility theory and other methods of modeling consumer preferences are emphasized. The appropriateness of von Neumann-Morgenstern utility theory to consumer choice is discussed and an empirical application of modeling a consumer utility function for new health services is presented.

Hausner M. 1954. Multidimensional utilities. In: Thrall RM, Coombs CH, Davis RL, eds. Decision processes. New York: Wiley, p. 167.

The von Neumann and Morgenstern theory of utility is generalized by omitting the Archimedean postulate. Refined methods are introduced to simplify the work and extend it to the infinite dimensional case. Two distinct concepts are considered: the set of prospects, called the mixture space, and the ordering or utility on this set. Axioms for both concepts are discussed. It is felt that the two set of axioms, taken together, characterize a utility space. A weak utility space is also considered. The weakened axioms are deemed strong enough to permit identification of indifferent elements and still preserve the operations of mixture and order. Results indicate that a mixture space may be embedded in a vector space and that a utility space may be embedded in an ordered vector space. Ordered vector spaces are examined.

Hoepfl RT et al. 1970. A study of self-explicated utility models.
Behav. Sci. 15:408.

Two types of utility models were compared on the basis of their ability to predict the subjects' evaluations of hypothetical university faculty members. The models obtained using mutiple regression were better predictors than the models obtained using a parameter-eliciting questionnaire, as expected, but the differences were small. An analysis of variance suggested that increasing the number of factors included in the descriptions caused the subjects to be less able to make consistent evaluations even in the small range included in the study, and also suggested that subjects who were required to respond to fewer factors in early trials and more factors in later trials made less consistent evaluations overall than subjects who were required to respond to more factors in early trials and fewer factors in later trials. The data also suggested that the subjects thought that the factors had a more uniform impact on the evaluations than was actually the case. It is concluded that self-explicated utility models have predictive validity in situations where the important variables have no important interactions and can be categorized into mutually exclusive levels within mutually exclusive factors. More information is deemed necessary to determine the superiority of these models over pure regression or hybrid rating-regression models.

Howell WC et al. 1971. On the allocation of inference functions in decision systems.
Org. Beh. Human Perform. 6:132.

Information processing and decision performance by a hierarchal system was studied under the realistic circumstance in which some input data are processed intuitively and some are processed by a man-machine (PIP) combine, the objective being to determine how much use the system is able to make of information that must be added to the PIP aggregate "by hand". Three ranges of such unanticipated data were introduced with two procedures for handling it: before and after PIP arrived at its opinion (i.e., as an adjustment in the priors for PIP, and as a revision in the PIP posteriors). From a total of 1,722 problems, it is concluded that performance improves with manually aggregated data, but that it asymptotes after four to six such items. It is felt that this implies a limited human capacity. Order of processing is deemed to have little or no effect.

Huber GP. 1968. Multiplicative utility models in cost effectiveness analysis. J. Ind. Eng. 19:17.

The improvement of cost effectiveness analysis as an aid in decision making is demonstrated by regarding a key step, the development of a quantitative measure of effectiveness, as the development of a general multiplicative utility model. An example problem illustrates this proposal.

Huber GP. 1974. Methods for quantifying subjective probabilities and multi-attribute utilities.
Decision Sci. 5:430.

Methods are described for eliciting subjective probabilities and multiattribute utilities whose usefulness has been empirically studied and reported in the research literature. The variable interval method, the fixed interval method, the equivalent prior sample method, the hypothetical future sample method, and betting and wagering are analyzed for eliciting subjective probabilities. Client-explicated and observer-derived models are examined for eliciting multiattribute utilities. Guidelines for the elicitation and use of such judgements are also included.

Huber GP. 1974. Multi-attribute utility models: A review of field and field-like studies.
Manage. Sci. 20:1393.

Published field and field-like research studies concerned with the development and use of multi-attribute utility models are reviewed with the particular needs of the management scientist in mind. The studies reviewed include only those conducted under conditions where uncertainties were not of practical importance. In addition, the experimental subjects were all occupationally experienced in making the utility judgments required of them. The findings from the research dealing with additive and multiplicative models are discussed and methods for estimating the parameters of each are presented, along with the advantages and disadvantages of these methods. It is concluded that behavioral science research validated the relatively recent management science practice of using subjective values as parameters in multi-attribute utility functions. It is also observed that, using as a choice criterion the

ability of the model to predict actual decisions and evaluations, simple additive models usually do as well as more complex additive models, about as well as the conjunctive multiplicative model, and better that the disjunctive multiplicative models. Although individual studies sometimes resulted in the finding that a particular model form is more predictive for a particular client-problem combination, the differences in performance are deemed small from a practical point of view.

Huber GP et al. 1969. A study of subjective evaluation models. Behav. Sci. 14:483.

To identify the difficulties that arise when staff groups must obtain a quantitative representation of an individual's or an organization's evaluation scheme, the ability of professional hospital personnel to develop and use a subjective evaluation model in a reliable manner was studied. The assigned task was to evaluate twelve hypothetical hospital wards, described in terms of seven quantitive factors; linear, addilog, and multiplicative models were used to approximate the resulting evaluation models. Results indicate that while professionals can develop and use a reliable subjective evaluation model, they are not very effective in perceiving and/or using the organizational evaluation scheme. A lack of similarity among their evaluation models was observed. It is also noted that earlier arguments supporting the use of addilog or multiplicative model forms are not particularly relevant in this experimental situation, based on a comparison of the magnitudes of the multiple correlations and other significant values. Reasons are offered for this finding.

Humphreys P. 1977. Application of multi-attribute utility theory. In: Jungermann H, de Zeeuw G, eds. Decision making and change in human affairs. Dordrecht, Holland: Reidel: p. 165.

Multiattribute utility theory (MAUT) is evaluated in terms of its claim to success in providing an axiomatization of decomposition and recomposition of complex phenomena which is both coherent and didactic. MAUT is placed in context as part of a multilevel axiomatic system which affords a decomposition of processes underlying choice behavior through a theoretically infinite

number of levels; however, only the three levels of acts, choice alternatives, and multiattributed outcomes for riskless choice are descibed, along with various solutions which can be adopted in applications of MAUT when assumptions underlying the axiomatizations are found to be violated in practice. The mapping of raw data obtained in applications of MAUT into a form suitable for input to a MAUT composition algorithm is considered, and mapping-plus-composition algorithms commonly proposed for use in MAUT-appropriate situations are evaluated. Three approaches to the validation of MAUT are discussed. Problems involved in the future development of dynamic multiattribute utility theory are presented.

Humphreys P et al. 1975. An investigation of subjective preference orderings for multi-attributed alternatives. In: Wendt D, Vlek C, eds. Utility, probability, and human decision making. Dordrecht, Holland: Reidel: p. 119.

Development is presented of a meaningful set of descriptors linking the Raiffa analysis of preference for multi-attributed alternatives with the Kelly "repertory grid" technique for the elicitation of attribute dimensions. Providing the assumptions underlying the repertory grid were reasonably upheld, it was found that people's actual preference orderings were closely replicated by preference orderings predicted by a technique developed from Raiffa's model. The basis for such replication was examined by comparing the accuracy of predictions based on the Raiffa analysis with those based on several other analyses which did not incorporate the complete set of operations described by the Raiffa model. These comparisons indicated that: (1) models for deriving unidimensional preference orderings from multidimensional data which do not take utilities into account were unsuccessful; (2) techniques for converting multidimensional representations of utility into preference orderings based on additive rules yielded considerably better results than did a selection rule; and (3) given the adoption of an additive model, the use of a lottery technique for assessing the relative value-wise importance of the various attribute dimensions resulted, in most cases, in an improvement of the predictive efficiency of the model over models which assign equal weights to all dimensions. Two limitations on the generality of these conclusions are also noted.

Johnsen E. 1976. Experiences in multiobjective management processes. In: Zeleney M, ed. Multiple criteria decision making, Kyoto 1975. New York: Springer-Verlag, p. 135.

Conclusions are reported from experiences in multiobjective problem solving at the individual, group, organizational, and national levels. Solution approaches combined managerial and problem solving roles to facilitate a mutual search and learning process. Different problems are analyzed with respect to an individual's or a group's reaction to them; group interactions in pursuit of a collective goal also are examined. It is concluded that: (1) people are normally unaware of their objectives but do need them in the managerial process; (2) people are willing and able to cope with objective problems as a link in a normal problem solving process; (3) objectives change all the time; (4) objectives become meaningful only in a process in which they interact with means; and (5) the objective-setting process never stops.

Johnson EM et al. 1977. The technology of utility assessment. IEEE Trans. on Sys. Man Cyber. SMC-7:311.

Utility assessment is defined, the utility assessment process is reviewed, and selected issues and problems are discussed. The focus is on applications in field and field-like settings where it is necessary, in order to solve a problem, to assess the utility of items or alternatives having more than one valued property. Although some aspects of the paper are new, it is noted that much of the work is tutorial in nature. Assessment techniques are discussed with particular emphasis on the factors which determine the relative effectiveness of the various techniques in specific situations. Assessment models also are discussed, along with a description of the areas where they have been applied. Guidelines are developed for improving the probability of success in applying utility assessment, and suggestions are made for the research necessary to improve further the technology of utility assessment.

Kasubek W et al. 1978. Optimierung subjektiver urteile: anwendung der multiattributen nutzentheorie bei medizinischen therapieentscheidungen. Zeitschrift fur experimentelle und angewandte Psychlogie 25:594.

The usefulness of multiattribute utility theory in improving medical therapy decisions is demonstrated with respect to the use of cortisone for treatment of certain respiratory illnesses. The seven most used cortisone drugs were evaluated. Five experienced physicians estimated utility functions and importance weights for the six side effects which were used as attributes. An indifference curve method and a conditional rating procedure were used. Results show that the physician's multiattribute judgments agree nearly perfectly and much more than their intuitive global evaluations of the alternatives.

Keeley SM et al. 1972. Bayesian and regression modeling of graduate admission policy.
Org. Beh. Human Perform. 8:297.

Bayes' theorem and multiple regression analysis are compared as descriptive models of four faculty judgments of 528 hypothetical graduate school applicants. Both approaches are conceptualized within lens model terminology. All four judges weighted grade point average very heavily and showed relatively similar weighting strategies. Number of cues did not have a major effect on cue utilization, but cue inconsistency did. Both the Bayesian model and the multiple regression model were found to provide reasonably good descriptions of the overall jugdments, with the regression model accounting for more of the judgment variance in all cases. Assuming conditional independence, aggregation was observed to be non-optimal, the judgments generally being conservative. The simple linear model proved an excellent descriptive model of three of the judges, and a configural model described the fourth judge very well. Some fruitful hypotheses for explaining the discrepancy between man's policy and Bayesian policy were provided by captured policies. It is argued that comparing Bayesian and regression models on the same data may prove fruitful, especially in understanding deviations from optimality.

Keeney RL. 1971. Utility independence and preference for multiattributed consequences.
Oper. Res. 19:875.

The concept of utility independence is defined and general expressions are derived for simplifying the assessment of multiattribute utility functions, given that certain utility independence assumptions hold. These expressions are presented as representation theorems; the proofs for them are given and the additive utility function is shown to be a special case. The usefulness of certainty equivalents in evaluating multiattributed utility preferences in uncertain situations is discussed. The relevance of the work is considered as an approximation technique when requisite assumptions are not satisfied.

Keeney RL. 1968. Quasi-separable utility functions.
Naval Res. Log. Quart. 15:551.

A method of assessment of utility functions of multi-numeraire consequences is presented. Emphasis is restricted to the case of two numeraires, where utility is non-decreasing in one numeraire and non-increasing in the other. Based on von Neumann and Morgenstern axioms of rational behavior and two additional assumptions, an equation is derived for a quasi-separable utility function of two-numeraire consequences. Implications and ramifications of such a utility function and its requisite assumptions are discussed. A technique for practical application of this work is presented. It is felt that quasi-separable utility functions are more generally applicable and less restrictive than separable utility functions, and only slightly more difficult to use.

Keeney RL. 1973. A decision analysis with multiple objectives: The Mexico City airport.
Bell J. Econ. Manage. Sci. 4:101.

Results are reported of a study done for the Ministry of Public Works of Mexico to help select the most effective strategy for developing the airport facilities of the Mexico City metropolitan area to insure quality air service for the remainder of the century. The decision analytic model used to evaluate strategies is described. The attributes of cost, safety, capacity of

airport facilities, noise levels, social disruption, and access times were adapted to account for impacts over time, and probability density functions and a utility function were assessed over the six attributes. Details of these assessments are given. Results and implications of the analyses are discussed. It is noted that a unique aspect of the study involved the assessment of a multiattribute utility function and its use to aid an important policy decision.

Keeney RL. 1973. Risk independence and multiattributed utility functions. Econometrica 41:27.

Conditional risk aversion, the conditional risk premium, and risk independence pertaining to multiattributed utility functions are defined. The functional form of the two-attribute utility function satisfying certain reasonable assumptions concerning an individual's conditional risk aversion attitudes is derived. The notion of risk independence is then generalized and renamed utility independence. Two representation theorems are stated which simplify the assessment of multiattributed utility functions, provided that specified risk independence and utility independence assumptions hold.

Keeney RL. 1974. Multiplicative utility functions.
Oper. Res. 22:22.

Sufficient conditions for a multiattribute utility function to be either multiplicative or additive are presented. It is stated that the number of requisite assumptions to imply the main result is equal to the number of attributes. Because the assumptions involve only trade-offs between two attributes at a time or lotteries over one attribute, it is deemed reasonable to expect that decision makers can ascertain whether these assumptions are appropriate for their specific problems. Procedures are given for verifying the assumptions and assessing the resulting utility functions. Application of the results to a six-attribute problem relating to the development of Mexico City's airport facilities is sketched.

Keeney RL. 1976. A group preference axiomatization with cardinal utility. Manage. Sci. 23:140.

The group decision problem is formulated whereby the group preference and the individual preferences are utilities of the alternatives in the von Neumann-Morgenstern sense. Two models are investigated: (1) the certain alternative model, where it is assumed that alternatives have known consequences; and (2) the uncertain alternative model, where there may be uncertainties associated with the consequences of the decision. For both models, five reasonable assumptions are postulated, and it is shown that the group cardinal utility function which is implied must be a linear combination of the individual cardinal utility functions. It is noted that the rules explicitly require interpersonal comparison of preferences. Suggestions for obtaining the necessary assessments to utilize these aggregation rules are given.

Keeney RL. 1972. Multiplicative utility functions.
Tech. Rep. 70, Cambridge, Mass: Mass. Inst. Tech.

Necessary and sufficient conditions for a multiattributed utility function to be either multiplicative or additive are presented. It is shown that the additive form is a limiting case of the multiplicative form, and it is observed that the number of requisite assumptions to imply the main result is equal to the number of attributes. Because the assumptions involve only trade-offs between two attributes at a time or lotteries over one attribute, it is deemed reasonable to expect that decision makers can ascertain if the assumptions are appropriate for their specific problem. Procedures are given for verifying the assumptions and assessing the resulting utility functions. The application of the results to a six-attribute problem relating to the development of Mexico City's airport facilities is briefly discussed.

Keeney RL. 1972. Utility functions for multiattributed consequences. Manage. Sci. 18:276.

The restrictiveness of existing techniques for systematically assessing multiattributed utility functions valid for decision making under uncertainty is addressed. Utility independence and conditional utility functions are defined, and operational assumptions about the decision maker's preferences

for multiattributed consequences are postulated. General functional forms are then derived which the two-attribute and n-attribute utility functions must satisfy subject to the attributes being mutually utility independent. In both cases, the assessment of the multiattributed utility function is greatly simplified. The resulting functional forms are deemed more general than the additive and log-additive utility functions discussed in the brief survey of previous work on multiattributed utility functions. A procedure is suggested for verifying whether an attribute is utility independent of the others in a particular situation.

Keeney RL. 1977. The art of assessing multiattribute utility functions. Org. Beh. Human Perform. 19:267.

The appropriateness of multiattribute utility theory (MAUT) for developing preference models to address value trade-offs among multiple objectives and uncertainty in complex problems is demonstrated by applying MAUT to a specific problem involving energy policy. An 11-attribute utility function over attributes including deaths, sulfur dioxide pollution, radioactive waste, health effects, and electrical energy generated is assessed. A dialog indicating the procedure used, with comments on why various questions were asked, is presented in detail. It is noted that the resulting utility function is being used to examine energy policies differing in terms of main fuel (fossil or nuclear) and degree of conservation.

Keeney RL et al. 1975. Group decision making using cardinal social welfare functions.
Manage. Sci. 22:430.

A formal method for group decision making is suggested which explicitly addresses two complexities often present in group decision problems: (1) uncertainty exists concerning the exact impact of each of the alternatives; and (2) the individuals have different preference attitudes toward risks. The group Bayesian position is accepted for analyzing two extreme types of decision problems. In the first, a single person or a small subgroup makes the decision; in the second, the entire group is responsible for the decision. Representation theorems are then developed which provide for the

functional form of a social welfare function whose arguments are the individual utility functions of the group members. The results are discussed for different types of group decisions. Related work is briefly reviewed.

Keeney RL et al. 1977. An illustrative example of the use of multiattribute utility theory for water resource planning.
Water Resour. Res. 13:705.

Five alternative plans for developing the water resources of the Tisza River basin in Hungary, involving consideration of economic, environmental, social, and technical objectives, are evaluated. Twelve attributes are defined to indicate the degree to which the objectives are achieved in this project. A preliminary multiattribute utility function is assessed over these attributes. This is combined with existing information describing the possible consequences of the five plans to yield an overall rating of their desirability. The utility function was found explicitly to indicate the preference trade-offs among the attributes. Discussion suggests further uses of the utility function in the planning and evaluation processes.

Kirkwood CW. 1976. Parametrically dependent preferences for multiattributed consequences.
Oper. Res. 24:92.

The concept of parametric dependence of preferences is introduced to assist in assessing multiattributed consequences. The usefulness of the concept for the assessment of multiattribute utility functions is shown by proving several representation theorems that simplify the assessment problem when parametric dependence holds. Operational problems related to the application of the theorems in practical situations are also discussed.

Kirkwood CW. 1978. Social decision analysis using multiattribute utility theory. In: Zionts S, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p.335.

Incorporation of the preferences of different groups into a decision analysis for public sector decision making is examined. A conflict between Pareto optimality and certain equity considerations is identified. It is

shown that there is no way to incorporate preferences in such a situation which is guaranteed to be both efficient, in the sense that no other solution will make some group better off without making another worse off, and equitable, in the sense of considering the different impacts on various groups. In addition, practical difficulties in applications work are discussed, and areas for future research are indicated.

Klahr D. 1969. Decision making in a complex environment: The use of similarity judgements to predict preferences.
Manage. Sci. 15:595.

Judgments of the relative similarity of pairs of alternatives are used to construct a model of the decision space of a group of college admissions officers. The model is then used to predict the preferences of the officers for a set of hypothetical college applicants. The accuracy of the predictions supports the hypothesis that preference judgements are made on the basis of the similarity of given alternatives to an ideal alternative. A nonmetric multidimensional scaling procedure is used to construct the space. This procedure yields a dimensional representation based upon very few assumptions about the nature of the similarity measures.

Klee AJ. 1971. The role of decision models in the evaluation of competing environmental health alternatives.
Manage. Sci. 18:B-52.

Problems faced by the Solid Waste Management Office in evaluating competing environmental health alternatives, given a set of decision criteria, are discussed. The concepts of scoring models are reviewed, and techniques for implementing a linear scoring model or pairwise comparisons are critically examined. The Decision Alternative Ratio Evaluation (DARE) method is then introduced. Using a sample problem of dismantling retired wooden railroad cars, it is shown that the DARE method is based upon a pairwise comparison concept but that it drastically restricts the total number of comparisons involved. Use of DARE in conjunction with other rating methods is suggested for further simplifying the decision making process.

Kornbluth JSH. 1978. Ranking with multiple objectives. In: Zionts S, ed. Multiple criteria problem solving. Berlin: Springer-Verlag, p. 345.

An interactive method for ranking items subject to an initially unspecified linear utility function is presented. The method is economical on the number of paired judgments that must be made by the decision maker and leads to the identification of the desired ranking and the space of weights for the corresponding linear utility functions which would lead to this ranking. Simulation results suggest that in cases where the number of attributes is six or less, a maximum of 20 to 30 items must be considered in order to obtain a reasonable estimate of the appropriate weight space. It is felt that the method is useful in any situation where either the rank order or the associated utility weights (or both) are of interest. The method can also be used as an aid in the analysis of decision making, finding appropriate weights or weight intervals for further multi-criterion optimization, and as an alternative approach to interrogative techniques for evaluating the preferences of a decision maker.

Kunreuther H. 1969. Extensions of Bowman's theory on managerial decision-making.
Manage. Sci. 15:B-415.

The validity of Bowman's theory concerning the quality of managerial decision making is evaluated against the three general criterial of consistency over time, economic significance, and statistical significance. These criteria are then used to evaluate a production planning model for an electronics firm where the rules are based on average past behavior. The production plans based on an average rule are compared with the actual plans for the company. It was found that when a manager has limited information regarding future sales, as when he develops his initial production estimates, then a plan based on average behavior performs better than actual initial plans. However, when environmental cues provide reliable information on future sales of specific items (as when revisions are made), then actual decisions are clearly superior to those suggested by an average rule. It is felt that this case study offers insight into the potential usefulness as well as the limitations of Bowman's hypothesis. Suggestions for future research are offered.

Lincoln DR et al. 1979. Cross-media environmental impacts of coal-fired power plants: An approach using multi-attribute utility theory. IEEE Trans. Sys. Man Cybern. SMC-9:285.

A human preference model, developed using multiattribute utility theory, was linked to an environmental emission model for a coal-fired power plant to develop a methodology for identifying optimal control strategies reflecting specific pollutant trade-off decisions by environmental decision makers. Six emissions to air, water, and land were evaluated. It was found that the pollutant most desired to be reduced was either sulfur dioxide or particulates to the air. For solid wastes, most individuals tested preferred reducing sludge rather than ash to minimum levels; heat emitted to air received the lowest rating from all individuals. It is stressed that the results do not represent a comprehensive analysis because individuals were asked for their environmental preferences only, and other important factors such as cost were not considered

Luce RD. 1956. Semiorders and a theory of utility discrimination. Econometrica 24:178.

Imperfect response sensitivity to small changes in utility is examined in relationship to intransitivities of the indifference relation. A comparatively simple theory of utility is constructed in which an individual is not sensitive to all changes in utility; the theory yields a nonstatistical analog of the "just noticeable difference" concept of psychophysics. Next, a set of preference axioms which allow for intransitive indifference relations is introduced, and relations satisfying these axioms are termed "semiorders." The set of intransitive indifference relations is shown to be substantially equivalent to a utility theory in which there are just noticeable difference functions which state the change in utility for any value of utility so that the change is just noticeable. The precise form of the function is examined in detail in the case of risk represented by a linear utility function over a mixture space.

Luce RD. 1958. A probabilistic theory of utility.
Econometrica 26:193.

A model for choices among uncertain alternatives is developed in which preferences between pure alternatives and likelihood judgements between events are assumed to be independent probabilistic processes. It is noted that this model is, in some respects, a probabilistic version of von Neumann-Morgenstern utility models. Certain plausible notions about subjective probability are shown to imply a very simple discrimination function between events in which the probability of choice depends only upon differences of subjective probabilities. Similarly, the expected utility hypothesis is shown to imply that preference discrimination depends upon utility differences, and the form of that discrimination function is determined. Questions of empirical verification are discussed.

MacCrimmon KR. 1968. Decision making among multiple-attribute alternatives: A survey and consolidation approach.
Memorandum RM-4823-ARPA, Rand Corporation, Santa Monica, California.

Several methods and techniques for making quantitative and qualitative evaluations between multiple attribute alternatives are examined. The methods discussed include dominance, satisficing, maximim, maximax, lexicography, additive weighting, effectiveness index, utility theory, trade-offs, and non-metric scaling. The assumptions underlying each approach are critically reviewed, as are the information requirements of each. In addition, each method is described in a general way and by using a formal, abstract mathematical representation. Two examples, the choice of a weapons system and of a space suit, illustrate the discussion. It is observed that most theoretical discussions of decision making methods assume that the exact values of the attributes are known when, in fact, such information is seldom known with such precision. It is then noted that this survey discusses situations where there is uncertainty about attribute values, presenting a combination of methods which may be used as a more reasonable and valid approach than the choice of any single method.

MacCrimmon KR. 1973. An overview of multiple objective decision making. In: Cochrane JL, Zeleny M, eds. Multiple criteria decision making. Columbia, S.C.: Univ. S. C. Press, p. 18.

The feasibility of combining methods when dealing with multiple objective and multiple attribute decisions is investigated. A meta-model is developed to relate the multiple objectives of a decision maker and the multiple attributes characterizing his alternatives to the models an analyst builds of these processes and choices. This serves as a framework for examining a variety of prescriptive and descriptive models grouped under the headings of weighting methods, sequential elimination methods, mathematical programming methods, and spatial proximity methods. Each method is briefly described by its most prominent features and by its underlying assumptions; a representative example of each model's use is also presented. As a basis for developing effective combinations of methods, the conditions appropriate for using each model are indicated through an extended example.

Madden JM. An application to job evaluation of a policy-capturing model for analyzing individual and group judgement.
Tech. Rep. PRL-TDR-63-15, Lackland Air Force Base, Texas: Personnel Research Lab.

A multiple regression model combined with a mathematical grouping model is presented as a method of estimating individual rater consistency and degree of interrater agreement in developing a job evaluation plan. Officers ranked 50 simulated Air Force specialities, each of which consisted of pre-assigned scale values for 10 job requirement factors. Thirty-eight officers ranked the jobs on the basis of merited grade, and 36 officers ranked the jobs on merited pay. Each rater's consistency was evaluated by a multiple regression equation predicting his rank ordering of the jobs from the factor values. Consistency of policy among raters was measured by the loss in predictive efficiency when a single equation represented the joint policy of the group. Measures of rater consistency showed that all but two of the raters were adequately consistent. Measures of interrater agreement indicated that raters were applying a homogeneous policy, whether they ranked on merited pay or merited grade. The officer raters were found to be capable of applying a consistent policy in evaluating jobs when their only information was an estimate of the job requirements.

May KO. 1954. Intransitivity, utility, and the aggregation of preference patterns.
Econometrica 22:1.

Some experimental evidence and theoretical arguments are presented suggesting that neither cardinal nor ordinal utility theory can reflect, even to an approximation, the existing preferences of individuals in many economic situations. The argument is based on the necessity of transitivity for utility, observed circularities of preferences, and a theoretical framework that predicts circularities in the presence of preferences based on numerous criteria.

Miller JR. 1967. A systematic procedure for assessing the worth of complex alternatives.
AD 662001, Defense Documentation Center.

An explicit, logically consistent, and replicable procedure for assessing the worth of complex alternative is developed. The procedure assumes that a decision context has been specified and that a fixed set of discrete alternatives has been chosen. It then assesses the worth of each alternative, estimates the resource drain required by each, and combines these considerations, along with considerations of risk and uncertainty, to arrive at a final decision. Results of an experiment designed to measure the impact of this procedure upon professional decision makers also are reported. To place the procedure in perspective, it is compared with alternative procedures discussed in the literature and practiced by decision makers.

Newman JR. 1975. Assessing the reliability and validity of multi-attribute utility procedures: An application of the theory of generalizability.
SSRI Res. Rep. 75-7, Los Angeles: Univ. Southern Calif.

A theoretical rationale is presented for assessing the reliability, validity, and dependability of multiattribute utility models and techniques. A study of generalizability is conducted by taking measurements assumed to be randomly representative of the universe to which the investigator wishes to generalize. The ratio of an estimate of the universe score variance to an estimate of the observed score variance is defined as the coefficient of

generalizability. It is estimated by the intra-class correlation coefficient. Analysis of variance and an expected mean square paradigm are used to obtain the appropriate variance estimates. It is felt that the theory dispenses with unnecessary and unwarranted assumptions and eliminates the distinction between reliability and validity. It is further felt that any generalizability study can be conducted without reference to a parallel measure of the multiattribute utility (MAU) instrument or some external criterion of success. If a MAU technique is compared to a non-MAU technique for doing the same thing, it is possible to calculate the coefficient of generalizability for both models and choose the method best suited to one's needs. Three examples are given for this theory. Preliminary investigations are believed to indicate that MAU models, and techniques based on such models, may be better than non-MAU models since the former have a tendency to reduce the interation between judges and the item being judged when such interation represents inconsistency of judgement.

Pekelman D et al. 1974. Mathematical programming models for the determination of attribute weights.
Manage. Sci. 20:127.

Several versions of a mathematical programming model which determines attribute weights for each consumer are empirically evaluated using data on dry cereals and automobiles. The model uses a linear compensatory model incorporating an ideal point concept and is found to be a good predictor of consumer preferences, particularly for dry cereals. The model also indicated that most consumers use very few attributes in evaluating brands, suggesting that it is feasible to segment the market in terms of determinant attributes. Managerial implications for product repositioning, new product design, and market segmentation are then discussed.

Polak E. 1976. On the approximation of solutions to multiple criteria decision making problems. In: Zeleny M, ed. Multiple criteria decision making, Kyoto 1975. New York: Springer-Verlag, p. 271.

An adaptive precision method is presented based on cubic splines and a new scalarization procedure which constructs approximations to the surface of noninferior points. The method is felt to be particularly well-suited for

two-criteria and three-criteria optimization problems and applicable to some extent to high dimensional problems. The technique is considered efficient since it computes no more points than necessary to ensure a prescribed level of precision of approximation.

Pollak RA. 1967. Additive von Neumann-Morgenstern utility functions. Econometrica 35:485.

A necessary and sufficient condition for the additivity of von Neumann-Morgenstern utility functions is presented. A "strong additivity axiom" is introduced, and it is shown that an individual's preferences satisfy this axiom if and only if his von Neumann-Morgenstern utility function is additive. Next, a "weak additivity axiom" is presented, and it is demonstrated that an individual's preferences satisfy it if and only if his von Neumann-Morgenstern utility function is either additive or log additive. Restrictions resulting from these assumptions about the von Neumann-Morgenstern utility function are discussed. The extension of these results to the continuous consumption case is indicated.

Richard SF. 1975. Multivariate risk aversion, utility independence and separable utility functions.
Manage. Sci. 22:12.

The concept of multivariate risk aversion is introduced as a pairwise property which a decision maker may evidence when making decisions concerning some pairs of variables but not when making decisions concerning other pairs. Multivariate risk-averse behavior, risk-neutral behavior, and risk-seeking behavior are examined. Next, a multivariate risk premium is defined and sufficient conditions are given for one multivariate risk averse utility function to have larger multivariate risk premiums than another. Operations preserving multivariate risk aversion are proved, and all definitions and theorems are generalized to utility functions for n-attributes. Finally, certain assumptions are given which are necessary and sufficient for a multivariate utility function to have one of three possible separable forms. It is shown that the negative multiplicative form has multivariate risk aversion, while the positive multiplicative form and the additive form are risk-seeking and risk-neutral, repectively.

Roy B. 1973. How outranking relation helps multiple criteria decision making. In: Cochrane JL, Zeleny M, eds. Multiple criteria decision making. Columbia, S.C.: Univ. of S. C. Press, p. 179.

The concept of an outranking relation $S_A$ is developed as a binary relation defined on a set A of feasible decisions. Implications of the definition are then studied with regard to the transitivity of the binary relation $S_A$ and to the stability of $S_A$ when A is enriched with some new decisions. Practical ways of building outranking relations also are considered. The use of outranking relations is examined in different kinds of decision problems to: (1) rough out a problem by eliminating a large subset of elements of A when one and only one decision must be selected; (2) build a trichotomy of A including decisions accepted, rejected, or those requiring a complement of information; and (3) obtain a weak order on A.

Shepard RN. 1964. On subjectively optimum selection among multiattribute alternatives. In: Shelley MW, Bryan GL, eds. Human judgement and optimality. New York: Wiley, p.257.

Techniques are examined to help an individual achieve his subjectively given subgoals and to reduce the occurrence of subsequent judgements that the wrong choice was made in some earlier decision. Two aspects of this situation are identified. In the first, the failure to achieve a subgoal is attributable to limitations on an individual's power of rational calculation. In the second, the conflict between subgoals is considered as resulting from the number of objectively disparate attributes that the subgoals possess. The suggestion is then put forward of having individuals and computers share in the decision making process. Elementary comparisons with respect to the underlying subjective attributes would be performed by an individual, while the purely logical or combinatorial process of putting these elementary judgements together to reach an overall decision would be entrusted to a computer. An experiment then illustrates that the relative weights to be assigned to component attributes are not always determineable and may depend on the adoption of one of several incompatible but equally tenable systems of subjective goals. It is hypothesized that the tendency to accept a spurious resolution of a problem is caused by temporarily accepting a special state of

mind or an unrealistic goal that has the advantage of entailing a system of weights that clearly favors one alternative over its competitors and permits the decision to be consummated.

Slovic P. 1969. Analyzing the expert judge: A descriptive study of a stockbroker's decision processes.
J. Appl. Psychol. 53:255.

An analysis-of variance technique is illustrated for describing the use of information by persons making complex judgments. The subjects were two stockbrokers who rated the growth potential of stocks on the basis of all factors taken from Standard and Poor's reports. The technique proved capable of providing a precise quantitative description of configural and nonconfigural information utilization. It was found that each broker exhibited a substantial amount of configural processing. The method is felt to have promise for providing experts with insight into their own processes and for teaching and evaluating student judges.

Slovic P et al. 1971. Comparison of bayesian and regression approaches to the study of information processing in judgement.
Org. Beh. Human Perform. 6:649.

A review and comparative analysis is presented of the Bayesian and regression approaches to the study of information utilization in judgement and decision making. For each approach the following items are examined: (1) models developed for describing and prescribing the use of information in decision making; (2) major experimental paradigms, including the types of judgement, prediction, and decision tasks and the kinds of information that have been available to the decision maker in these tasks; (3) key independent variables that have been manipulated in experimental studies; and (4) major empirical results and conclusions. The two approaches are compared to determine, first, if the specific models and methods characteristic of different paradigms direct the researcher's attention to certain problems and cause him to neglect others that may be equally important and, secondly, to learn if a researcher studying a particular substantive problem can increase his understanding by employing diverse models and experimental methods.

Slovic P et al. 1972. Analyzing the use of information in investment decision making: A methodological proposal.
J. Business 45:283.

Techniques for analyzing the cognitive processes involved in decision making are investigated. Complex computer simulations, linear models, and configural models are briefly reviewed and the analysis of variance (ANOVA) technique is explained. An experiment involving broker's and student's stock predictions is used to illustrate the ANOVA model. Results suggest that ANOVA and multiple regression analysis show promise as devices for describing the use of information in investment decisions. Factors determining which method would be more suitable in a given situation are also discussed.

Slovic P et al. 1977. Behavioral decision theory.
Ann. Rev. Psychol. 28:1.

Normative and descriptive facets of behavioral decision theory are reviewed. Descriptive studies of judgement, inference, and choice are considered first, and then the development of decision-aiding techniques is discussed. Several trends in the literature are noted. First, it is observed that while decision making is being studied by researchers in diverse disciplines, the importance of psychological concepts in decision making is increasing in both normative and descriptive work. Second, increasing effort is seen being devoted to the development of practical methods for helping people cope with uncertainty. Third, the number of reviews and bibliographies appearing in recent years is felt to indicate that the field of decision theory is growing rapidly. It is then noted that the present review focuses on psychological aspects of individual judgement and decision making.

Smith CG. 1979. Consultation and decision processes in a research and development laboratory.
Admin. Sci. Quart. 15:203.

A study of the relationship between consultation and decision processes in an industrial research laboratory showed the efficacy of multidirectional consultation coupled with a pattern of shared, decentralized decision making. The loose, decentralized pattern is closely associated with more

science-oriented activity, while the hierarchical pattern is associated with more practical or organizationally relevant activities. The relationships obtained between either the loose or the hierarchical pattern and performance are found to be mediated by coordination, adequacy of work expectations, and levels of member involvement. The results also suggest that when decision centers are consistent with consultation centers, implying a congruence between authority and expertise, the overall structure of the laboratory is less important.

Stimson DH. 1977. Utility measurement in public health decision making. Manage. Sci. 16:B-17.

The utilities of objectives of decision makers in a large public health agency were assessed using the Churchman-Ackoff approximation measure of value. The values obtained formed part of a model of a resource allocation problem faced by the agency in allocating a federal grant to improve the out-of-hospital care of the chronically ill and aged. A comparision of the normative solution derived from the model with the solution already chosen by the agency tended to support the hypothesis that agency members chose among alternatives as if they were maximizing expected utility.

Swezey RW. 1979. An application of a multi-attribute utilites model to training analysis.
Hum. Factors 21:183.

A multiattribute utilities model is applied to the design of improved gunnery ranges for a military anti-armor training system. Results of the application are presented and the model is then compared to a second simple judgement analysis model using the same input data. It was found that the two methods agreed on five of the six top-ranked attributes. Aspects of the multiattribute utilities approach are discussed.

Thrall RM. 1954. Applications of multidimensional utility theory. In: Thrall RM, Coombs CH, Davis RL, eds. Decision processes. New York: Wiley, p.181.

Some non-Archimedean utility space axioms are presented with special attention given to the consequences of weakening certain of them. Next, several applications of non-Archimedean utilities are discussed. It is noted that non-Archimedean utilities are perfectly satisfactory for game theory. The equivalence of game theory and linear programming guarantees that non-Archimedean utilities will be satisfactory also for linear programming problems.

Turban E et al. 1971. Utility theory applied to multivariable system effectiveness evaluation.
Manage. Sci. 17:B-817.

Utility and decision theories are applied to determine the effectiveness of a complex system with several measures of performance. Systems analysts and operational personnel in the Naval Air Development Center and related organizations were asked to rank the measurements of performance according to their importance and to assign utility indices to each. Churchman and Ackoff's method for checking the consistency of the ranking was modified and applied to the study. The evaluations of the individual judges were then adjusted and the estimates of all the judges were combined to form an overall utility index. The "coefficient of concordance" test was used to measure the degree of agreement among the judges. The weighted overall measure of performance is used to evaluate proposed courses of action and to establish stopping rules for a simulation study.

von Winterfeld D. 1975. An overview, integration, and evaluation of utility theory for decision analysis.
SSRI Res. Rep. 75-9. Los Angeles: Univ. Southern Calif.

Measurement theoretic literature on utility models and assessment is surveyed. The role of utility theory as a part of the general theory of measurement is explained first, and a classification scheme for utility models is developed. Models are classified according to some of their formal

properties and to the type of decision situation to which they apply. Next, the main utility models of weak order measurement, difference measurement, bisymmetric measurement, conjoint measurement, and expected utility measurement are described through their assumptions, model forms, formally justified assessment procedures, and common approximation methods. Similarities and differences among models and assessment procedures are then discussed. Topics include logical relationships between models, similarities in the cognitive processes involved in the different assessment procedures, and model convergence by insensitivity. Finally, the use of utility theory for decision analysis is evaluated as a tool in formal treatments of decision problems. It is concluded that utility theory can be useful in structuring evaluation problems and in eliciting appropriate model forms, but that the theoretically feasible assessment procedures are often too clumsy and complicated to be applicable in real world preference assessment. A general critique of current trends to mathematize utility theory concludes the report.

von Winterfeldt D et al. 1973. Evaluation of complex stimuli using multi-attribute utility procedures.
Tech. Rep. 011313-2-T. Ann Arbor, Mich.: Engineering Psych. Lab., Univ. of Mich.

The validity and the usefulness of multiattribute utility theory (MAUT) models and procedures for evaluating complex stimuli were investigated in an apartment evaluation problem. After the construction of utility functions and importance weights, the subjects were trained to apply the evaluation strategy with which they felt most comfortable and which reflected their preferences best. The correlation between model predictions and complex judgements after training constituted a general measure of the usefulness of MAUT. Three procedures to assess utility functions and importance weights were tested for their differential validity beyond differences in the correlation with post-training judgements. Descriptions of apartments were constructed from weights and utility functions such that the three procedures would predict different preferences in simple choices. The three methods of assessing importance weights and utility functions were a probabilistic procedure, a direct rating procedure with importance weights derived from the

unstandardized utility functions, and a direct rating procedure with directly assessed importance weights. Results are deemed to indicate that MAUT can improve upon a decision maker's unaided intuition. The probabilistic procedure was found to be the superior method for predicting simple choices between stimuli.

von Winterfelt D et al. 1973. Multi-attribute utility theory: Models and assessment procedures.
Tech. Rep. 011313-7-T. Ann Arbor, Mich: Engineering Psych. Lab.

Multiattribute utility theory is reviewed from a measurement theoretic perspective. Decision situations are described and classified according to three salient aspects of choice: uncertainty, time-variability, and multi-dimensionality. For each choice situation the main mathematical representations, their interrelations, and differences are discussed. Measurement theoretic tests are described which separate between multiattribute utility models in riskless and risky time invariant choice situations. Assessment procedures are outlined to encode utility functions for the representations developed, and experimental applications of multiattribute utility theory are briefly reviewed.

von Winterfeldt D et al. 1975. Multi-attribute utility theory: Models and assessment procedures. In: Wendt D, Vlek CAJ, eds. Utility, probabilty and human decision making. Dordrecht, Holland: Reidel, p. 47.

This article is an exact duplicate of Technical Report 011313-7-T summarized above.

Wilkie WL et al. 1973. Issues in marketing's use of multi-attribute attitude models.
J. Market. Res. 10:428.

Results and conclusions of marketing research on the linear compensatory multiattribute attitude model are discussed. The basic compositional model is defined and a chart is constructed indentifying specific researches and issues. The issues are seen to include attributes, importance weight, beliefs, model structure, and model testing. Salient aspects of these issues

are then analyzed. It is felt that the review demonstrates the rapid advance from uncritical model application to a present atmosphere of controversy and cumulative analyses on specific issues. The review is also seen as showing the significance and difficulty of model formulations. Models that yield understanding and suggest feasible procedures for favorable control are recommended for future research.

Woods DH. 1966. Improving estimates that involve uncertainty.
Har. Bus. Rev. Vol. ?, p. 91.

The problems of bias and distortion which occur in the upward communication of judgements about crucial company sales and capital investments are examined. Specific reasons are cited for the occurrence of these problems and positive steps are offered to eliminate or reduce them. A capital budget format is proposed for incorporating information about uncertainty in a visible and precise manner. Relevant judgments are quantified at various points in the organization as they are passed upward so that a top-level manager has a better idea of what his operational managers and information specialists actually are thinking. In addition, the proposal to let individuals report the complete probability distribution of outcomes instead of forcing them to commit to a single estimate is seen as reducing bias because it allows operating managers to hedge their estimates and still communicate comprehensive information to the final approval level. Finally, it is suggested that top management communicate its actual preferences down to the expert who has to make the forecast.

Yilmaz MR. 1978. Multiattribute utility theory: A survey.
Theory and Decision 9:317.

Axiomatic developments in multiattribute utility theory, i.e., sets of preferential conditions that lead to multiattribute utility representations, are surveyed. Decomposition of multiattribute utilities is considered in situations involving riskless choice and decomposition under risk. It is noted that, in contrast with the former situation, decomposition involves preferences among probability distributions over a set of sure alternatives. Lexicographic utilities under certainty or risk are also covered, and results

are presented pertaining to the existence of group preferences and utilities. Some definitions and notation are included, as well as some standard results in unidimensional utility theory. Various empirical studies are excluded, such as experimental applications, assessment of utilities, or tests of preference axioms.

Yntema DB et al. 1965. Telling a computer how to evaluate multidimensional situations.
IEEE Trans. Hum. Factors Electron. HFE-6:3.

The feasibility of the "interpolation between corners" procedure as a psychophysical method for telling a machine how to compute the worths of multidimensional alternatives is demonstrated. A statistic called fractional disagreement is proposed as the proper measure of the machine's success in mimicking the man's decisions. The subjects were military officers who were asked to make decisions to which they could bring a large amount of experience. The fractional disagreement between a subject and the machine he had instructed was found to be, on the average, four percent, and the data suggest that the disagreement between subjects was of roughly the same size. The concept of conflict between the dimensions of two alternatives is also explained. The size of the conflict and the size of the difference between the worths that the machine ascribed to the alternatives were found to help predict the correctness of the machine's decision. It is concluded that conflict and difference in computed worth would be useful in defining regions where the machine would give the decision back to the man and tell him to make the choice. Some statistics and terminology that may be used in describing the performance of an evaluative decision maker are presented.

Zeleny M. 1975. MCDM Bibliography-(1975). In: Zeleny M, ed. Multiple criteria decision making, Kyoto 1975. New York: Springer-Verlag, p. 291.

A bibliography is presented which includes only published works directly related to multiple criteria decision making. Works excluded from the bibliography are those dealing with multidmensional scaling and conjoint measurement, multiple cues modeling of human judgment, multiattribute attitude models of consumer behavior, statistical decision theory, and similar topics.

It is noted that working papers and research memoranda appearing in regularly maintained institutional series are considered published materials for the purposes of this unofficial compilation.

Zeleny M. 1976. The theory of the displaced ideal. In: Zeleny M, ed. Multiple criteria decision making, Kyoto 1975. New York: Springer-Verlag, p.153.

An operational model is developed for analyzing the phenomenon of ideal displacement in the decision making process. Preferences are considered fully cardinal in character, though in a fuzzy and adaptive way. No indifference concept, no intransitivity assumption, and no axiom of the independence of irrelevent alternative apply. The paradigm is based on the ideal alternative, infeasible in general, providing the highest score with the respect to all individual attributes considered. The infeasibility of the ideal is observed to create a pre-decision conflict and generate the impulse to move as close as possible toward it, thus initiating adaptive information gathering and an evaluation process. Partial decisions are then made, inferior alternatives are removed, and post-decision dissonance is seen to ensue. Included in the decision making process are readjustment of attribute weights, displacement of the ideal, and new pre-decision conflict formation. The entire man-made interactive procedure is viewed as capable of leading to a decision in a finite number of iterations.